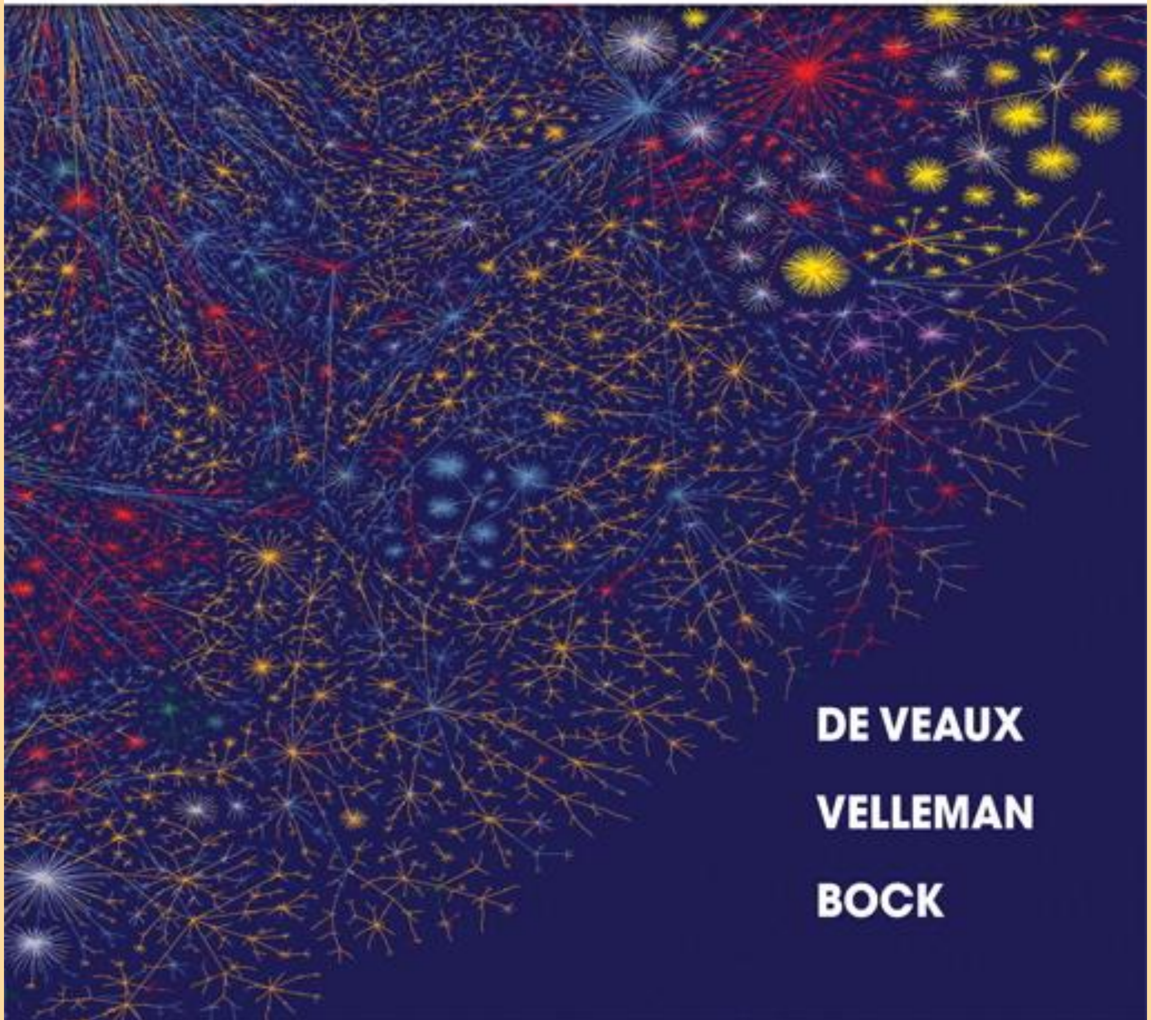
Third Edition

STATS Data and Models



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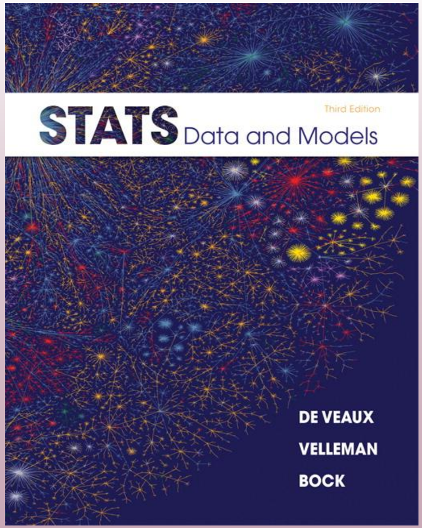
Chapter 10

Re-expressing Data:

Get it Straight!

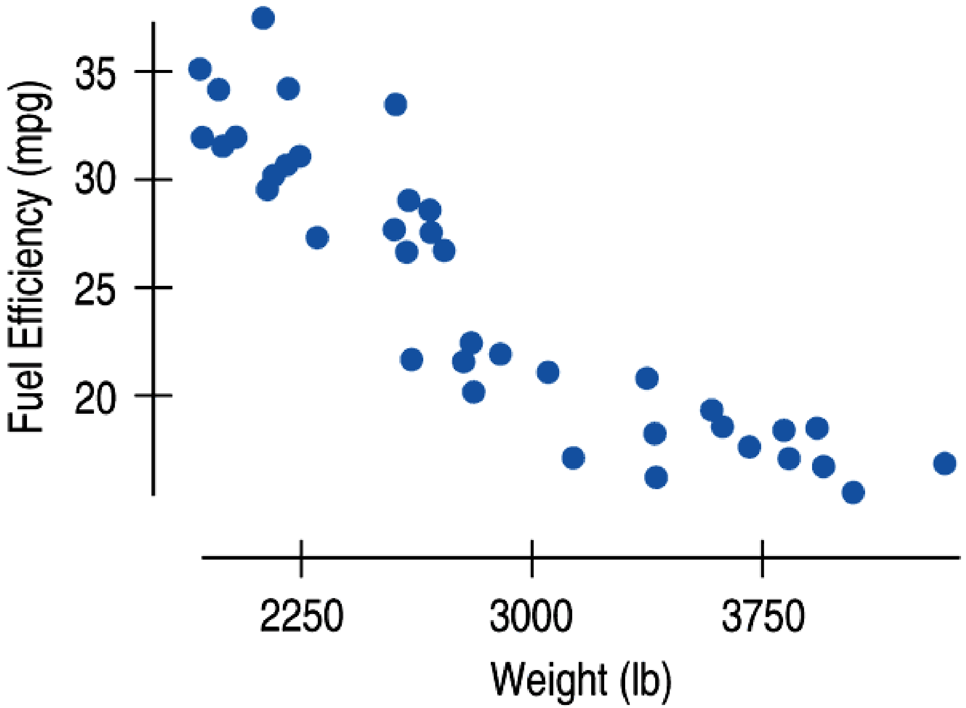


Straight to the Point

- We cannot use a linear model unless the relationship between the two variables is linear. Often re-expression can save the day, straightening bent relationships so that we can fit and use a simple linear model.
- Two simple ways to re-express data are with logarithms and reciprocals.
- Re-expressions can be seen in everyday life—everybody does it.

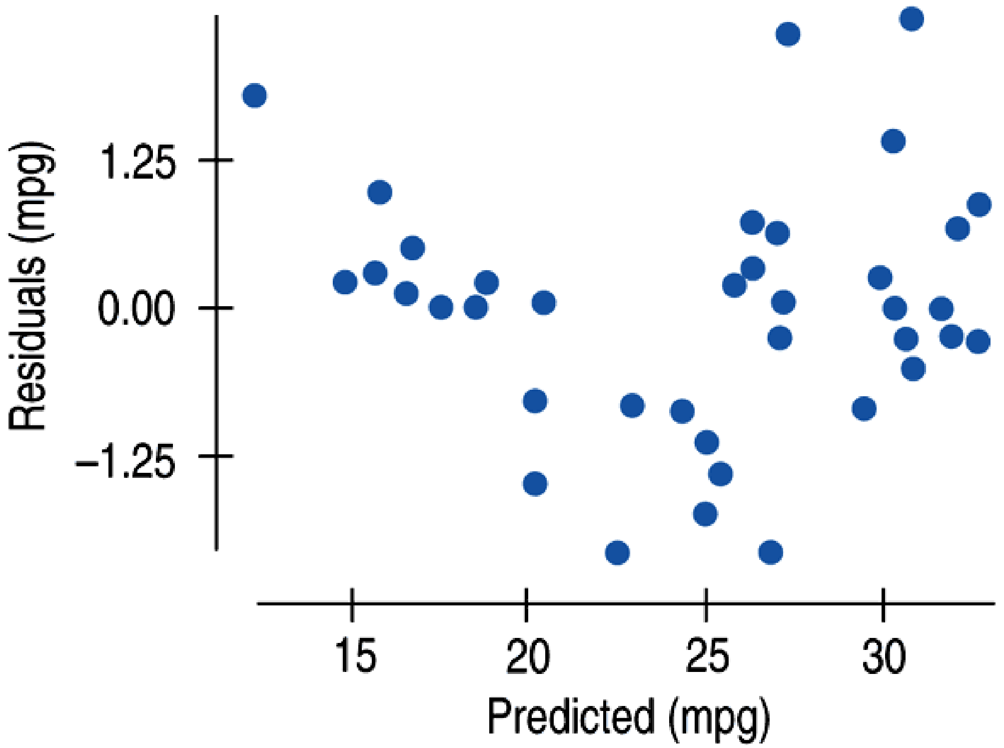
Straight to the Point (cont.)

- The relationship between *fuel efficiency* (in miles per gallon) and *weight* (in pounds) for late model cars looks fairly linear at first:



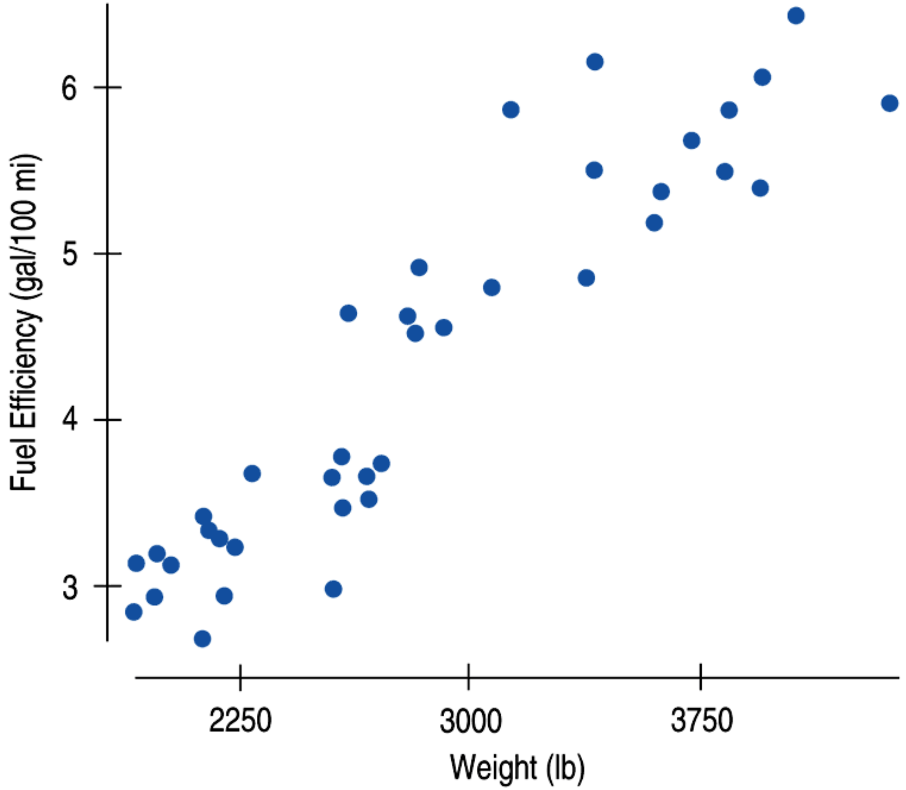
Straight to the Point (cont.)

- A look at the residuals plot shows a problem:



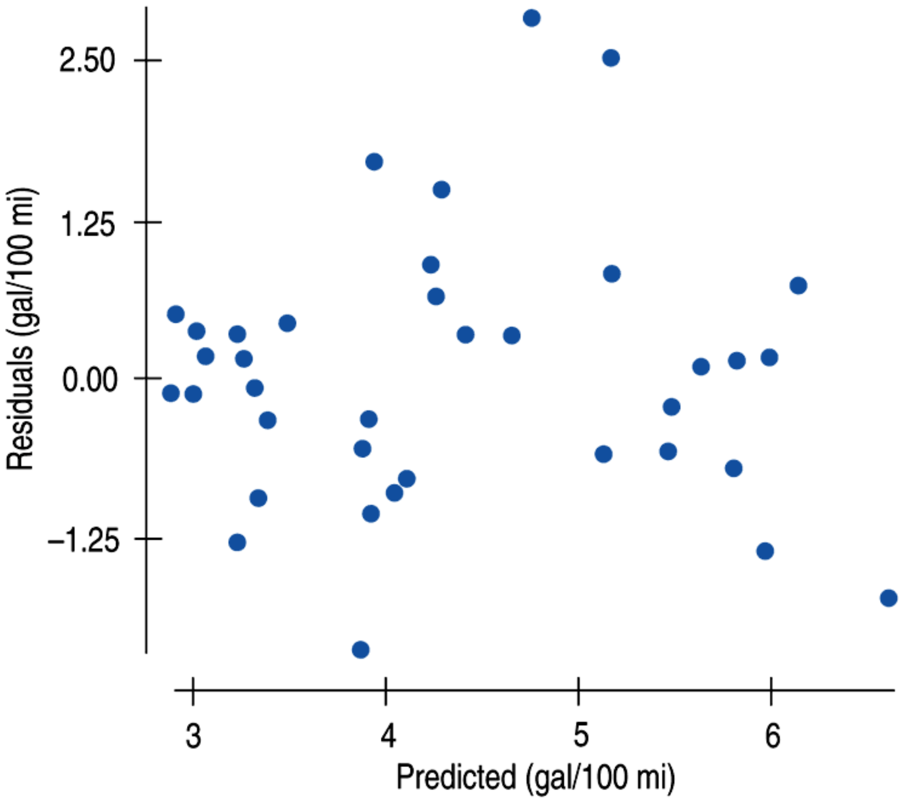
Straight to the Point (cont.)

- We can re-express *fuel efficiency* as gallons per hundred miles (a reciprocal) and eliminate the bend in the original scatterplot:



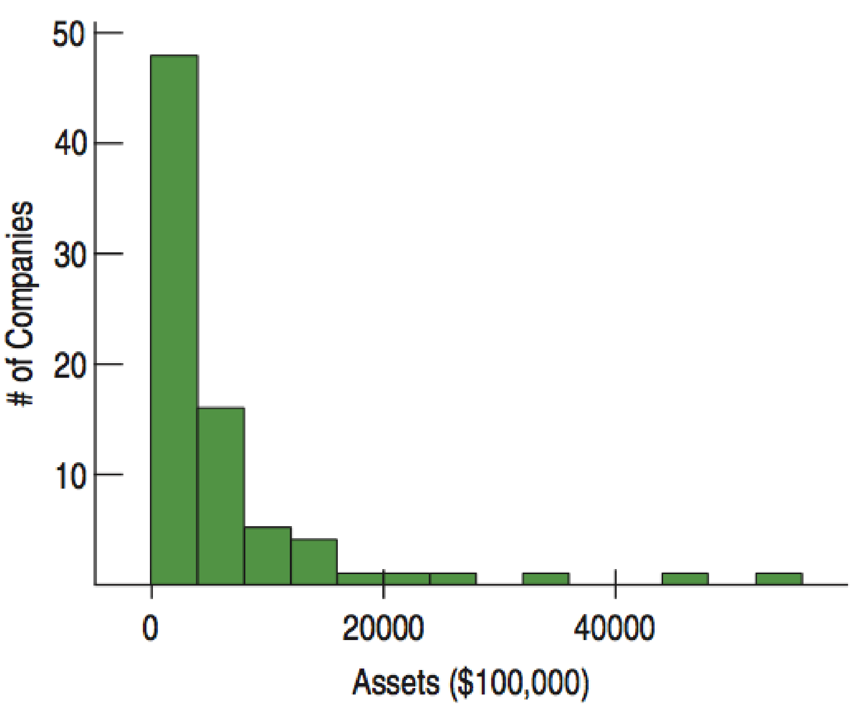
Straight to the Point (cont.)

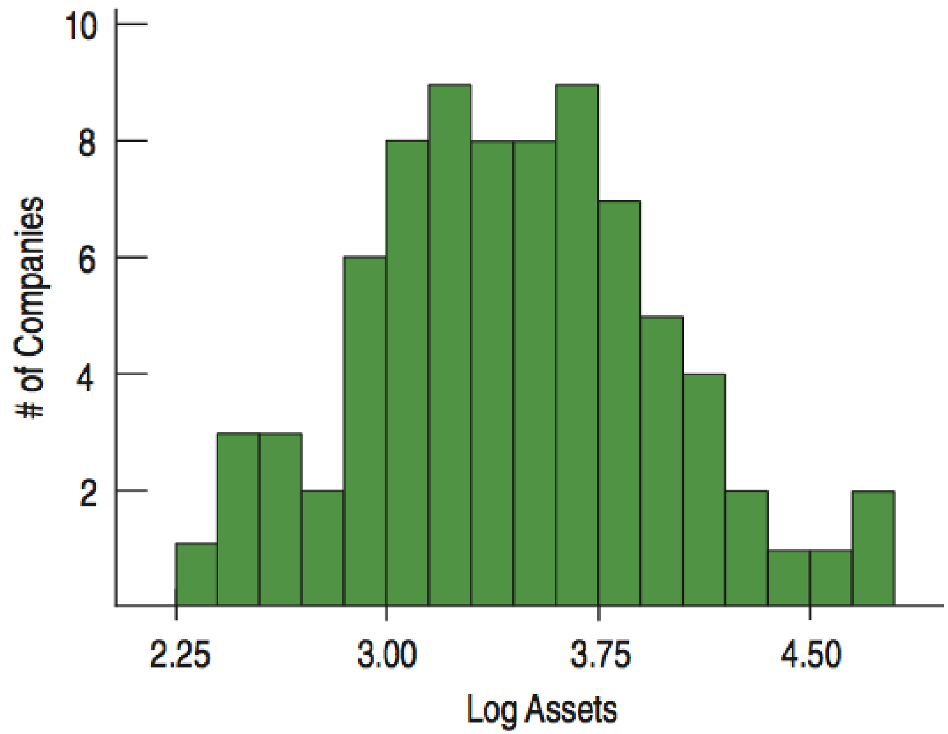
- A look at the residuals plot for the new model seems more reasonable:



Goals of Re-expression

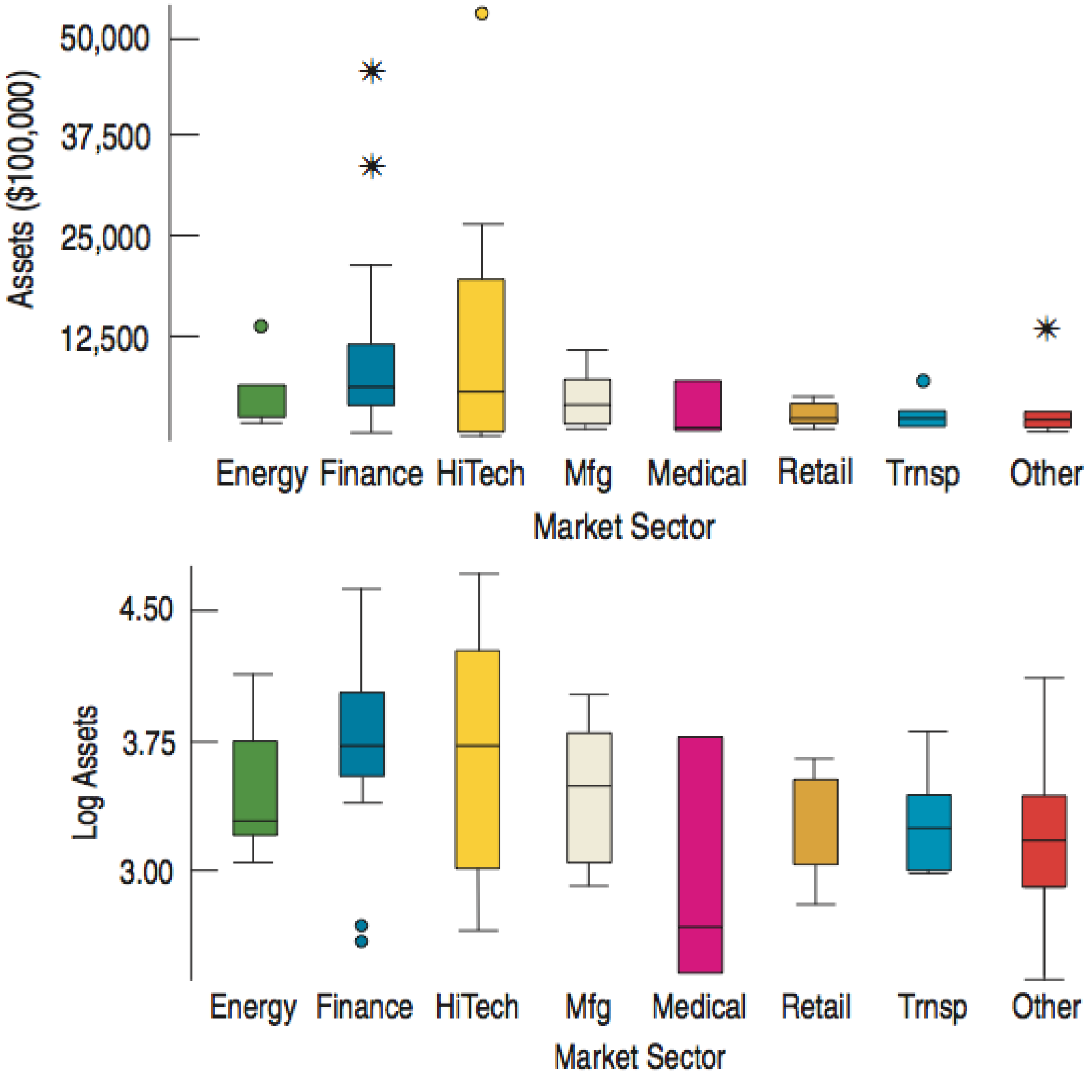
- Goal 1: Make the distribution of a variable (as seen in its histogram, for example) more symmetric.





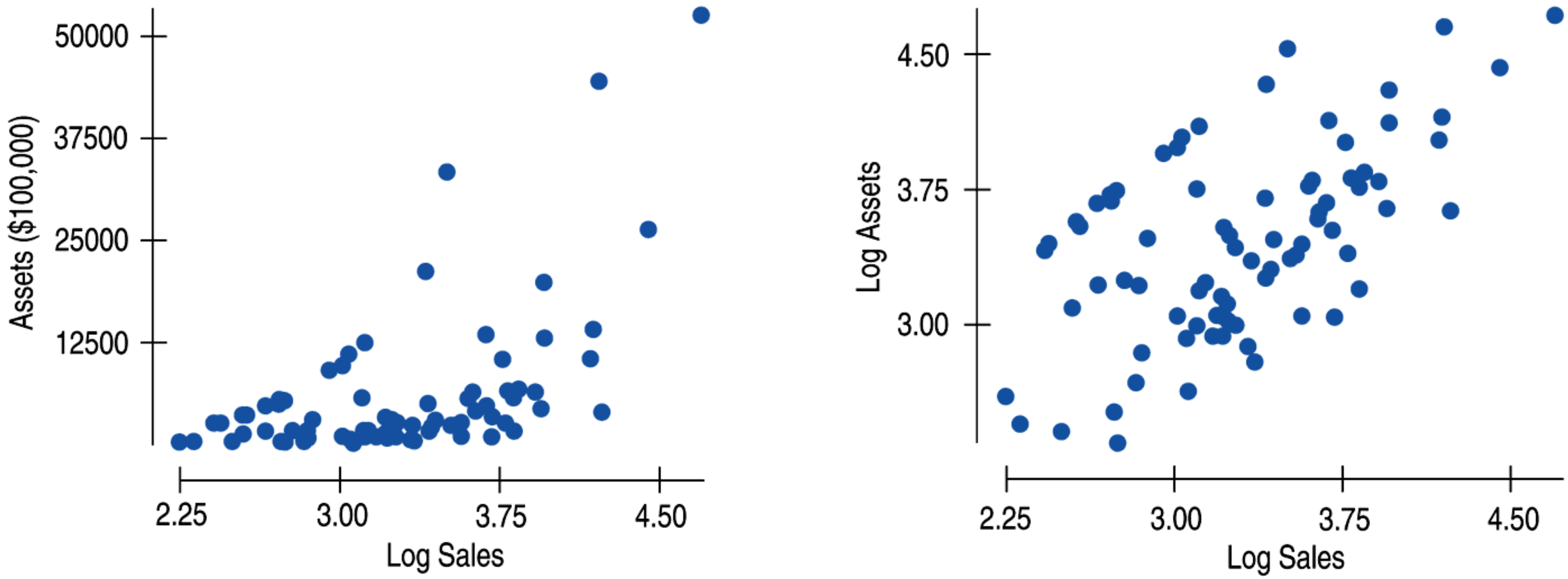
Goals of Re-expression (cont.)

- Goal 2: Make the spread of several groups (as seen in side-by-side boxplots) more alike, even if their centers differ.



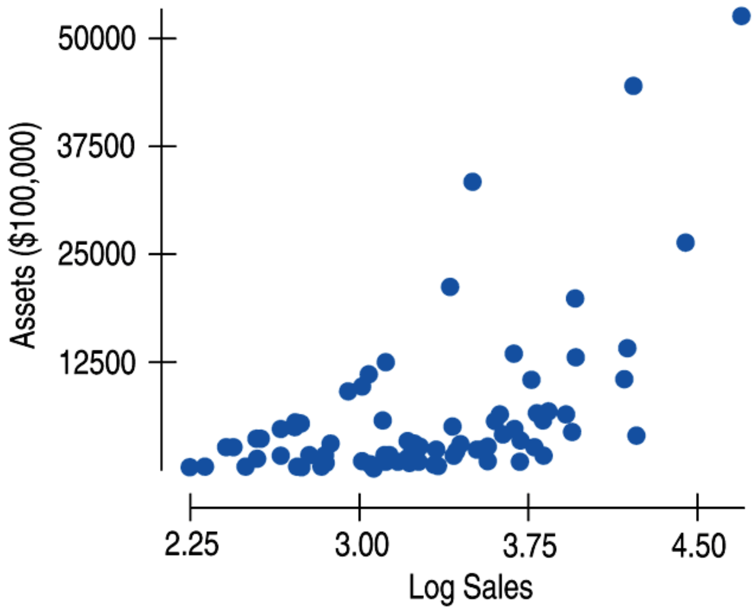
Goals of Re-expression (cont.)

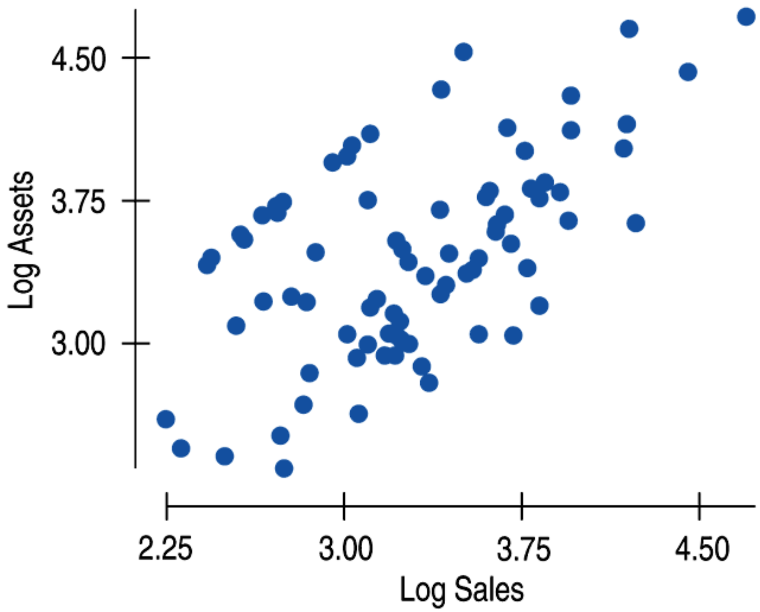
- Goal 3: Make the form of a scatterplot more nearly linear.



Goals of Re-expression (cont.)

- Goal 4: Make the scatter in a scatterplot spread out evenly rather than thickening at one end.
  - This can be seen in the two scatterplots we just saw with Goal 3:





The Ladder of Powers

- There is a family of simple re-expressions that move data toward our goals in a consistent way. This collection of re-expressions is called the Ladder of Powers.
- The Ladder of Powers orders the *effects* that the re-expressions have on data.

The Ladder of Powers

Power	Name	Comment
2	Square of data values	Try with unimodal distributions that are skewed to the left.
1	Raw data	Data with positive and negative values and no bounds are less likely to benefit from re-expression.
½	Square root of data values	Counts often benefit from a square root re-expression.
“0”	We’ll use logarithms here	Measurements that cannot be negative often benefit from a log re-expression.
–1/2	Reciprocal square root	An uncommon re-expression, but sometimes useful.
–1	The reciprocal of the data	Ratios of two quantities (e.g., mph) often benefit from a reciprocal.

Plan B: Attack of the Logarithms

- When none of the data values is zero or negative, logarithms can be a helpful ally in the search for a useful model.
- Try taking the logs of **both** the *x*- and *y*-variable.
- Then re-express the data using some combination of *x* or log(*x*) vs. *y* or log(*y*).

Plan B: Attack of the Logarithms (cont.)

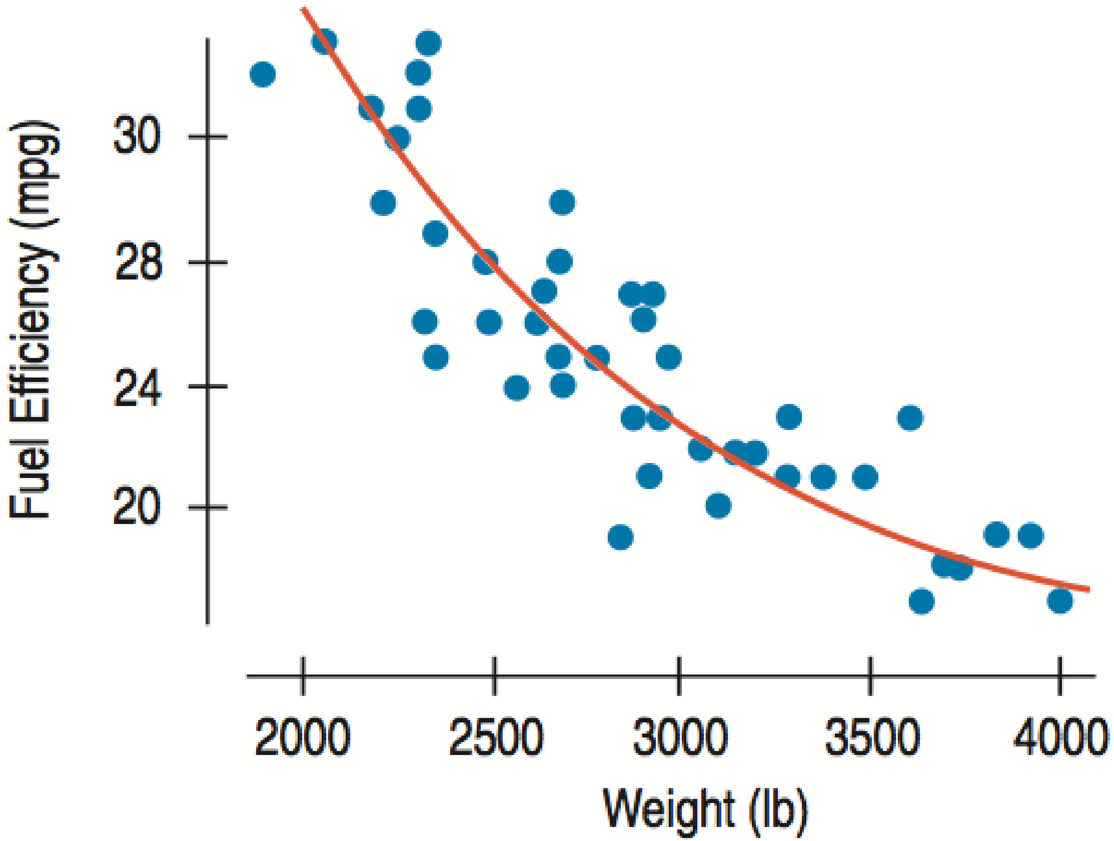
Model Name	*x*-axis	*y*-axis	Comment
Exponential	x	$\log(y)$	This model is the "0" power in the ladder approach, useful for values that grow by percentage increases.
Logarithmic	$\log(x)$	y	A wide range of x-values, or a scatterplot descending rapidly at the left but leveling off toward the right, may benefit from trying this model.
Power	$\log(x)$	$\log(y)$	The Goldilocks model: When one of the ladder's powers is too big and the next is too small, this one may be just right.

Multiple Benefits

- We often choose a re-expression for one reason and then discover that it has helped other aspects of an analysis.
- For example, a re-expression that makes a histogram more symmetric might also straighten a scatterplot or stabilize variance.

Why Not Just Use a Curve?

- If there's a curve in the scatterplot, why not just fit a curve to the data?



Why Not Just Use a Curve? (cont.)

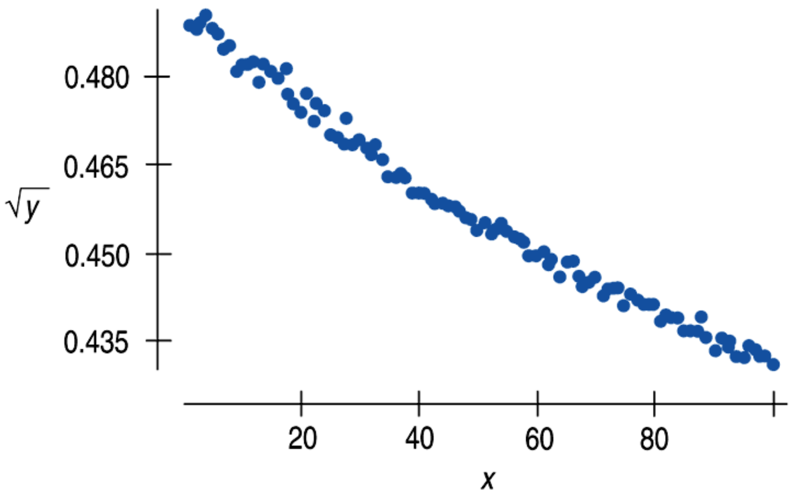
- The mathematics and calculations for “curves of best fit” are considerably more difficult than “lines of best fit.”
- Besides, straight lines are easy to understand.
  - We know how to think about the slope and the *y*-intercept.

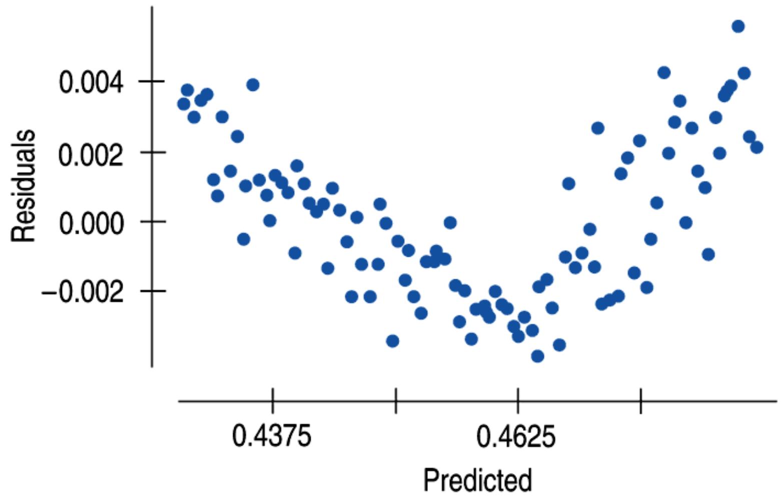
What Can Go Wrong?

- Don't expect your model to be perfect.

- Don't stray too far from the ladder.

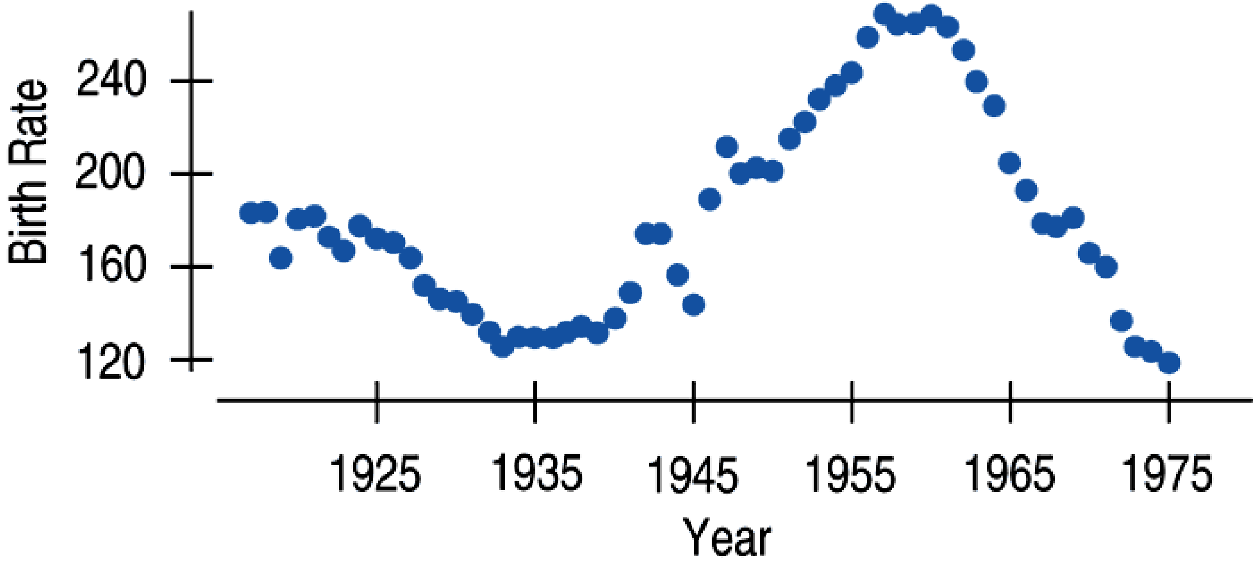
- Don't choose a model based on R^2 alone:





What Can Go Wrong? (cont.)

- Beware of multiple modes.
  - Re-expression cannot pull separate modes together.
- Watch out for scatterplots that turn around.
  - Re-expression can straighten many bent relationships, but not those that go up then down, or down then up.



What Can Go Wrong? (cont.)

- Watch out for negative data values.
  - It’s impossible to re-express negative values by any power that is not a whole number on the Ladder of Powers or to re-express values that are zero for negative powers.
- Watch for data far from 1.
  - Data values that are all very far from 1 may not be much affected by re-expression unless the range is very large. If all the data values are large (e.g., years), consider subtracting a constant to bring them back near 1.

What have we learned?

- When the conditions for regression are not met, a simple re-expression of the data may help.
- A re-expression may make the:
  - Distribution of a variable more symmetric.
  - Spread across different groups more similar.
  - Form of a scatterplot straighter.
  - Scatter around the line in a scatterplot more consistent.

What have we learned? (cont.)

- Taking logs is often a good, simple starting point.
  - To search further, the Ladder of Powers or the log-log approach can help us find a good re-expression.
- Our models won't be perfect, but re-expression can lead us to a useful model.